Pairing in High Temperature Superconductors and Berry Phase

D.Pal [1] and **B.Basu**[2]
Physics and Applied Mathematics Unit
Indian Statistical Institute
Calcutta-700035

Abstract

The topological approach to the understanding of pairing mechanism in high T_c superconductors analyses the relevance of the Berry phase factor in this context. This also gives the evidence for the pairing mechanism to be of magnetic origin.

The origin and nature of the high T_c superconductivity of the $La_{2-x}M_xCuO_{4-\delta}$ and $RBa_2Cu_3O_{7-\delta}$ compounds (M=Ba, Sr, Ca, Na .....and R=Y, La, Nd,...), as well as the other layered copper oxide compounds, is not presently understood and constitutes a formidable challenge to experimentalists and theorists alike. It is known that there is proximity of antiferromagnetism and superconductivity as the concentration of holes in the conducting CuO_2 planes is varied. This suggests the primary evidence for the pairing mechanism to be of magnetic origin. In this note we shall study the pairing mechanism through our understanding of high T_c superconductivity in terms of Berry phase as formulated in an earlier paper [1]. We shall show that a magnetic type of gauge interction is responsible for the formation of the pair leading to superconductivity.

In a recent paper [1] we have analysed the equivalence of Resonating Valence Bond (RVB) state with fractional quantum Hall fluid with filling factor $\nu = 1/2$ in terms of Berry phase which is associated with the chiral anomaly in 3+1 dimensions. It is noted that the three dimensional spinons and holons are characterized by the non-Abelian Berry phase and these reduce to 1/2 fractional statistics when the motion is confined to equatorial planes. We have shown that the topological mechanism of superconductivity is analogous to the topological aspects of fractional quantum Hall effect with $\nu = 1/2$. Our result corroborates with the idea of Laughlin [2a,2b]. In our framework it is argued that a frustrated spin system on a lattice is characterized by the chirality which is associated with the Berry phase factor μ where the phase is given by $e^{i2\pi\mu}$. We know that the ground state of an antiferromagnet is characterized by two operators, namely density of energy

$$\epsilon_{ij} = (\frac{1}{4} + \vec{S}_i.\vec{S}_j) \tag{1}$$

and chirality

$$W(C) = Tr \prod_{i \in C} (\frac{1}{2} + \vec{\sigma}.\vec{S}_i) \tag{2}$$

[1]e-mail : res9719@www.isical.ac.in
[2]e-mail : banasri@www.isical.ac.in

where σ are Pauli matrices and C is a lattice contour. Weigman [3] has related these operators with the amplitude and phase Δ_{ij} of Anderson's RVB through

$$\epsilon_{ij} = |\Delta_{ij}|^2 \tag{3}$$

$$W(C) = \prod_C \Delta_{ij} \tag{4}$$

This suggests that Δ_{ij} is a gauge field. The topological order parameter $W(C)$ acquires the form of a lattice Wilson loop

$$W(C) = e^{i\phi(c)} \tag{5}$$

which is associated with the flux of the RVB field

$$e^{i\phi(c)} = \prod_C e^{iA_{ij}} \tag{6}$$

A_{ij} is a phase of Δ_{ij} representing a magnetic flux which penetrates through a surface enclosed by the contour C. This phase is essentially the Berry phase related to chiral anomaly when we describe the system in three dimensions through the relation

$$W(C) = e^{i2\pi\mu} \tag{7}$$

where μ appears to be a monopole strength. In view of this, when a two dimensional frustrated spin system on a lattice resides on the surface of a three dimensional sphere of a large radius in a radial (monopole) magnetic field, we can associate the chirality with the Berry phase.

It is wellknown that the system of correlated electrons on a lattice is governed by the Hubbard model which in the strong coupling limit and at half filling can be mapped onto an antiferromagnetic Heisenberg model with nearest neighbour interaction which is represented by the Hamiltonian

$$H = J\sum(S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z) \tag{8}$$

with $J > 0$. As we have mentioned earlier, a frustrated spin system leading to RVB state is characterised by the chirality associated with it. In view of this we can consider an anisotropic Heisenberg Hamiltonian with nearest neighbour interaction which may be represented as

$$H = J\sum(S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z) \tag{9}$$

where $J > 0$ and $\Delta \geq 0$ and the anisotropy parameter Δ is given by $\Delta = \frac{2\mu+1}{2}$ [4], μ being the Berry phase factor and μ can take the values $\mu = 0, \pm 1/2, \pm 1, \pm 3/2........$. We note that for $\Delta = 1$ corresponding to $\mu = 1/2$ one has the isotropic antiferrromagnetic Heisenberg model. When $\Delta = 0 (\mu = -1/2)$ we have the XX model. It may be mentioned here that the relationship of the anisotropy parameter Δ with the Berry phase facter μ has been formulated from an analysis of the relationship between the conformal field theory in $1+1$ dimension, Chern-Simons theory in $2+1$ dimension and chiral anomaly in $3+1$ dimension [4].

Now for a two dimensional frustrated spin system on a lattice on the surface of a three dimensional sphere in a radial magnetic field the chirality demands that μ is non-zero

and should be given by $|\mu| = \frac{1}{2}$ in the ground state. However for $\mu = 1/2$, we get the isotropic antiferromagnetic Heisenberg Hamiltonian. Now let us consider the ground state of antiferromagnetic Heisenberg model on a lattice which allows frustration to occur giving rise to resonating valence bond (RVB) states corresponding to spin singlets where two nearest neighbour bonds are allowed to resonate among themselves. This situation occurs with $\mu = -1/2$ suggesting $\Delta = \frac{2\mu+1}{2} = 0$ in the Hamiltonian (9). But with $\Delta = 0$, the Hamiltonian effectively represents XX model corresponding to a bosonic system represented by singlets of spin pairs. This eventually leads to a resonating valence bond state giving rise to a nondegenerate quantum liquid. We have argued [1] that these spin singlet states forming the quantum liquid are equivalent to fractional quantum Hall (FQH) liquid with filling factor $\nu = 1/2$. Indeed, in earlier papers [5,6] we have pointed out that in QHE the external magnetic field causes the chiral symmetry breaking of the fermions (Hall particles) and an anomaly is realized in asssociation with the quantization of Hall conductivity. This helps us to study the behaviour of a quantum Hall fluid from the viewpoint of the Berry phase which is linked with chiral anomaly when we consider a 2D electron gas of N-particles on the surface of a three dimensional sphere in a radial (monopole) strong magnetic field. For the FQH state with $\nu = 1/2$ [5], the Dirac quantization condition $e\mu = 1/2$ suggests that $\mu = 1$. Then in the angular momentum relation for the motion of a charged particle in the field of a magnetic monopole

$$\vec{J} = \vec{r} \times \vec{p} - \mu \vec{r} \tag{10}$$

we note that for $\mu = 1$ (or an integer) we can use a transformation which effectively suggests that we can have a dynamical relation of the form

$$\vec{J} = \vec{r} \times \vec{p} - \mu \vec{r} = \vec{r'} \times \vec{p'} \tag{11}$$

This indicates that the Berry phase which is associated with μ may be unitarily removed to the dynamical phase. This implies that the average magnetic field may be taken to be vanishing in these states. However, the effect of the Berry phase may be observed when the state is split into a pair of electrons each with a constraint of representing the state with $\mu = \pm 1/2$. These pairs will give rise to the $SU(2)$ symmetry as we can consider the state of these two electrons as a $SU(2)$ doublet. This doublet of Hall particles for $\nu = 1/2$ FQH fluid may be considered to be equivalent to RVB singlets.

It is known that the RVB states are characterized by neutral spin $\frac{1}{2}$ excitations called spinons and charged spinless excitations called holons. To study these excitations in our Berry phase approach, we have to consider a system such that a two dimensional frustrated spin system lies on the surface of a three dimensional sphere in a radial (monopole) magnetic field which gives rise to the chirality associated with the monopole. As discussed above a RVB state may be characterised by associating a magnetic flux at the background corresponding to $\mu = -1/2$ in the Berry phase factor. Now if we consider a single spin down electron (with a $\mu = -1/2$ value) at a site j surrounded by an otherwise featureless spin liquid representing a RVB state the associated magnetic flux at the site j may be characterised by the monopole strength $\mu = -1/2$ which represents the Berry phase factor for the isolated spin down electron at the site j. Indeed, a spin polarised electron may be represented by a two component spinor such that spin down and up states correspond to

particle and antiparticle and from the relation $e\mu = 1/2$, we may associated spin down and up states with $\mu = -1/2$ and $+1/2$. Now the magnetic flux associated with the spin down electron at the site j when combined with the magnetic flux at the background due to the frustration of the spin system, we find the effective Berry phase factor is given by $\mu = -1$. This means that when a spinon moves in a closed path represented by a plaquette it will acquire the phase factor $|\mu| = 1$ for the Berry phase given by $e^{i2\pi\mu}$. The units of magnetic flux associated with the Berry phase factor μ may be represented through a phase

$$e^{i2\pi\mu} = \prod_C e^{iA_{ij}} \tag{12}$$

where A_{ij} is a phase representing a magnetic flux which penetrates through a surface enclosed by the contour C. As pointed out earlier, the Berry phase factor $|\mu| = 1$ effectively gives rise to a FQH state with $\nu = \frac{1}{2}$. Thus the neutral spin $\frac{1}{2}$ excitation, the spinon is characterized by $|\mu| = 1$ which may be split into two parts, with one spin $\frac{1}{2}$ excitation in the bulk and the other part is due to the *orbital spin* which is in the background characterized by the chirality of the frustrated spin system. The relation $e\mu = \frac{1}{2}$ suggests that the spinon effectively represents a particle having 1/2 fractional statistics in a 2D system which is analogous to the idea of Laughlin [2].

For the spin-charge recombination we have formulated a similar formalism of Weng, Sheng and Ting [7]. They have considered the spin-charge separation saddle-point by the effective Hamiltonian

$$H_{eff} = H_s + H_h \tag{13}$$

where H_s and H_h are the spinon and holon Hamiltonian respectively, defined by

$$H_s = -J_s \sum_{<ij>\sigma} (e^{i\sigma A^h_{ij}}) b^\dagger_{i\sigma} b_{j\sigma} + h.c \tag{14}$$

$$H_h = -t_h \sum_{<ij>} e^{i(-\phi^0_{ij} + A^s_{ij})} h^\dagger_i h_j + h.c \tag{15}$$

where $b_{i\sigma}$ and h_i are spinon and holon annihilation operators respectively. Here ϕ^0_{ij} represents flux quanta threading through each plaquette. The topological phases A^h_{ij} and A^s_{ij} are such that A^h_{ij} describes flux quanta attached to holons which are seen only by spinons in H_s and A^s_{ij} describes flux quanta bound to spinons which can only be seen by holons. Thus A^h_{ij} represents the doping effect into spin degrees of freedom where A^s_{ij} plays the role of a scattering source in holon transport.

In our framework, we can write the effective Hamiltonian for the spinon as

$$H_s = -J_s \sum_{<ij>\sigma} (e^{i\sigma A_{ij}}) b^\dagger_{i\sigma} b_{j\sigma} + h.c \tag{16}$$

where $b_{i\sigma}$ is the spinon annihilation operator and A_{ij} represents the magnetic flux penetrating through a surface enclosed by a contour C and is given by eqn.(12). Similarly we can write a Hamiltonian for holon

$$H_h = -t_h \sum_{<ij>} e^{i(-\phi^0_{ij} + A_{ij})} h^\dagger_i h_j + h.c \tag{17}$$

where $b_{i\sigma}$ and h_i are spinon and holon annihilation operators respectively. Here ϕ^0_{ij} represents flux quanta threading through each plaquette. The interaction between spinons and holons are then mediated through the gauge field A_{ij}.

When a hole is introduced, the spinon will interact with the hole through the propagation of the magnetic flux and this coupling will lead to the creation of the holon having magnetic flux corresponding to $|\mu_{eff}| = 1$. Eventually, the residual spinon will be devoid of any magnetic flux corresponding to $\mu_{eff} = 0$. This is realized when the unit of magnetic flux $\mu = -\frac{1}{2}$ associated with the single down spin in the RVB liquid will form a pair with another up spin having $\mu = +\frac{1}{2}$ associated with the hole. Thus the holon characterised by $|\mu_{eff}| = 1$ represent a spinless charged particle having $\frac{1}{2}$ fractional statistics in a 2D system. Indeed, the isolated spin in the RVB liquid will be combined with the spin associated with the hole when a spin pair is formed. Again, the holon having the effective Berry phase factor $|\mu_{eff}| = 1$ will also eventually form a pair of holes. This is because of the fact that as we see from eqn.(11), for any integer μ the Berry phase may be removed to the dynamical phase when the average magnetic field vanishes. The Berry phase is observed when the system forms a pair such that the units of magnetic flux are distributed among the pair. The spinons $s_{i\sigma}$ minimizes the energy by acquiring a pair amplitude given by

$$b_{ij} = s^\dagger_{i\uparrow} s^\dagger_{j\downarrow} - s^\dagger_{i\downarrow} s^\dagger_{j\uparrow} \tag{18}$$

Now we note that a spin pair each having unit magnetic flux and a pair of holes each having unit magnetic flux should interact with each other through the gauge field A_{ij} mentioned earlier.

It is now noted that the spin pairing as well as charge pairing in this scheme occurs through a gauge interaction. In case of spin pairing, we note that when the units of magnetic flux associated with the spinon having $|\mu_{eff}| = 1$ are transferred to the hole, the residual spinon having $|\mu_{eff}| = 0$ eventually forms a pair of spins having $\mu = 1/2$ and $-1/2$. The magnetic flux associated with each spin will give rise to a gauge field A_μ, lying in the link of the lattice, operating between them. Indeed, the Berry phase factor is associated with the chiral anomaly through the relation [8]

$$2\mu = -\frac{1}{2} \int \partial_\mu J^5_\mu d^4x \tag{19}$$

where J^5_μ is the axial vector current $\bar{\psi}\gamma_\mu\gamma_5\psi$. The association of a chiral current with spin is shown elsewhere [9]. When a chiral current interacts with a gauge field we have the anomaly given by

$$\partial_\mu J^5_\mu = Tr\ {}^*F_{\mu\nu} F_{\mu\nu} \tag{20}$$

where $F_{\mu\nu}$ is the field strength tensor of the gauge field A_μ. This field can be associated with the background magnetic field. Thus we may conclude that the gauge field A_{ij} which is of magnetic type is responsible for the spin-pairing observed in high-T_c superconductivity. The same view will also be valid for a pair of holes which is eventually formed when the holon gets its share of magnetic flux having $|\mu_{eff}| = 1$ from the spinon. This magnetic interaction is responsible for the hole pairing which is strong enough to overcome the bare Coulomb repulsion. This leads to the suggestion that the superconducting phase order will be established when a spin pair each having unit magnetic flux and a pair of holes

having unit magnetic flux interacts with each other through a gauge force. That is, the pair of holes will be attached to the spin pair such that spin-charge recombination occurs when each hole is attached to a spin site of the spin pair.

Also it is noted that the bosonic holon having $|\mu_{eff}| = 1$ and the residual *bosonic* spinon having $|\mu_{eff}| = 0$ (which eventually represents a pair) cannot give the correct statistics for electron when these two form a composite state. The correct statistics is only achieved when we introduce a phase associated with a unit of magnetic flux corresponding to $\mu = 1/2$ in this composite system. Thus the spinon holon recombination along with a phase shift only gives rise to an electron. This corroborates with the spin-charge separation description in a path integral formalism [10] where an electron is described as a composite particle of a spinon and holon together with a nonlocal phase-shift field.

From this approach, it appears that superconductivity and magnetism are closely related. Indeed spinon-holon interaction as well as the pair interaction is found to be of magnetic origin as the magnetic flux associated with the Berry phase is responsible for these features. Also we may infer about the phase diagram of the high T_c superconductiong states. As the number of holes are increased the number of spinon-holon recombined pair also increases, reaches a maximum at the optimally doped region. But as the percentage of the doped hole increases, in the overdoped region it may not find the spinon-pair to share its magnetic flux and the spin-charge recombination stops. This may explain the downfall of the phase diagram of high T_c superconductors in the overdoped region.

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